

# Production of doubly strange hypernuclei via $\Xi^-$ doorways in the $^{16}\text{O}(K^-, K^+)$ reaction at 1.8 GeV/c

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## Abstract

We examine theoretically production of doubly strange hypernuclei,  $^{16}_{\Xi^-}\text{C}$  and  $^{16}_{\Lambda\Lambda}\text{C}$ , in double-charge exchange  $^{16}\text{O}(K^-, K^+)$  reactions using a distorted-wave impulse approximation. The inclusive  $K^+$  spectrum at the incident momentum  $p_{K^-} = 1.8$  GeV/c and scattering angle  $\theta_{\text{lab}} = 0^\circ$  is estimated in a one-step mechanism,  $K^-p \rightarrow K^+\Xi^-$  via  $\Xi^-$  doorways caused by a  $\Xi^-p\Lambda\Lambda$  coupling. The calculated spectrum in the  $\Xi^-$  bound region indicates that the integrated cross sections are on the order of 7–12 nb/sr for significant  $1^-$  excited states with  $^{14}\text{C}(0^+, 2^+) \otimes s_{\Lambda p\Lambda}$  configurations in  $^{16}_{\Lambda\Lambda}\text{C}$  via the doorway states of the spin-stretched  $^{15}\text{N}(1/2^-, 3/2^-) \otimes s_{\Xi^-}$  in  $^{16}_{\Xi^-}\text{C}$  due to a high momentum transfer  $q_{\Xi^-} \simeq 400$  MeV/c. The  $\Xi^-$  admixture probabilities of these states are on the order of 5–9%. However, populations of the  $0^+$  ground state with  $^{14}\text{C}(0^+) \otimes s_{\Lambda}^2$  and the  $2^+$  excited state with  $^{14}\text{C}(2^+) \otimes s_{\Lambda}^2$  are very small. The sensitivity of the spectrum on the  $\Xi N\text{--}\Lambda\Lambda$  coupling strength enables us to extract the nature of  $\Xi N\text{--}\Lambda\Lambda$  dynamics in nuclei, and the nuclear  $(K^-, K^+)$  reaction can extend our knowledge of the  $S = -2$  world.

Keywords: Hypernuclei, DWIA,  $\Xi N\text{--}\Lambda\Lambda$  coupling

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## I. INTRODUCTION

It is important to understand properties of  $\Xi$  hypernuclei whose states are regarded as “doorways” to access multi-strangeness systems as well as a two-body  $\Xi N$ – $\Lambda\Lambda$  system, and it is a significant step to extend study of strange nuclear matter in hadron physics and astrophysics [1]. Because the  $\Xi$  hyperon in nuclei has to undergo a strong  $\Xi N \rightarrow \Lambda\Lambda$  decay, widths of  $\Xi$  hypernuclear states give us a clue to a mechanism of  $\Xi$  absorption processes in nuclei. A pioneer study of  $\Xi$  hypernuclei by Dover and Gal [2] has found that a  $\Xi$ -nucleus potential has a well depth of  $24 \pm 4$  MeV in the real part on the analysis of old emulsion data. However, our knowledge of these  $\Xi$ -nucleus systems is very limited due to the lack of the experimental data [3]. Indeed, the missing-mass spectra of a double-charge exchange (DCX) reaction ( $K^-$ ,  $K^+$ ) on a  $^{12}\text{C}$  target have suggested the  $\Xi$  well depth of 14–16 MeV [4, 5]. Several authors [6] have used the unsettled  $\Xi$ -nucleus (optical) potentials such as  $V_{\Xi} = (-24) - (-14)$  MeV and  $W_{\Xi} = (-6) - (-3)$  MeV in the Woods-Saxon potential to demonstrate the  $\Xi^-$  production spectra in the nuclear ( $K^-$ ,  $K^+$ ) reactions. There remains a full uncertainty about the nature of doubly strange ( $S = -2$ ) dynamics caused by the  $\Xi N$  and  $\Xi N$ – $\Lambda\Lambda$  interaction in nuclei at the present stage. More experimental information is earnestly desired.

The ( $K^-$ ,  $K^+$ ) reaction is one of the most promising ways of studying doubly strange systems such as  $\Xi^-$  hypernuclei for the forthcoming J-PARC experiments [3]. One expects that these experiments will confirm the existence of  $\Xi$  hypernuclei and establish properties of the  $\Xi$ -nucleus potential, e.g., binding energies and widths. This reaction can also populate a  $\Lambda\Lambda$  hypernucleus through a conventional DCX two-step mechanism as  $K^- p \rightarrow \pi^0 \Lambda$  followed by  $\pi^0 p \rightarrow K^+ \Lambda$  [7–9], as shown in Fig. 1(a). Such an inclusive  $K^-$  spectrum in the  $\Lambda\Lambda$  bound region is rather clean with much less background experimentally. Early theoretical predictions for two-step  $^{16}\text{O}(K^-, K^+)$  reactions at the incident momentum  $p_{K^-} = 1.1$  GeV/c and scattering angle  $\theta_{\text{lab}} = 0^\circ$  [7, 8] have indicated small cross sections for the  $\Lambda\Lambda$  states, for example,  $\sim 0.1$  nb/sr for the  $0^+(s_{\Lambda}^2)$  ground state and  $\sim 2$  nb/sr for the  $2^+(s_{\Lambda}^2)$  excited state in  $^{16}_{\Lambda\Lambda}\text{C}$  when we took 0.61 mb/sr and 0.32 mb/sr as the laboratory cross sections at  $0^\circ$  for  $K^- p \rightarrow \pi^0 \Lambda$  and  $\pi^0 p \rightarrow K^+ \Lambda$ , respectively.

It should be noticed that another exotic production of  $\Lambda\Lambda$  hypernuclei in the ( $K^-$ ,  $K^+$ ) reactions is a one-step mechanism,  $K^- p \rightarrow K^+ \Xi^-$  via  $\Xi^-$  doorways caused by a  $\Xi^- p \rightarrow \Lambda\Lambda$

transition, as shown in Fig. 1(b). The  $\Xi N$ - $\Lambda\Lambda$  coupling induces the  $\Xi^-$  admixture and the  $\Lambda\Lambda$  energy shift  $\Delta B_{\Lambda\Lambda} \equiv B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) - 2B_{\Lambda}({}_{\Lambda}^{A-1}Z)$  in the  $\Lambda\Lambda$ -nuclear states [10–14], and its coupling strength is also related to widths of  $\Xi$ -hypernuclear states [15, 16]. For a viewpoint of  $S = -2$  studies, it is very important to extract quantitative information concerning the  $\Xi N$ - $\Lambda\Lambda$  coupling from spectroscopy of the  $\Xi$  and  $\Lambda\Lambda$  hypernuclei [17, 18].

— FIG. 1 —

In this Letter, we study theoretically production of a doubly strange hypernucleus in the DCX ( $K^-$ ,  $K^+$ ) reaction on an  $^{16}\text{O}$  target at  $p_{K^-} = 1.8$  GeV/c and  $\theta_{\text{lab}} = 0^\circ$  within a distorted-wave impulse approximation (DWIA). Thus we focus on the  $\Lambda\Lambda$ - $\Xi$  spectrum for  ${}_{\Lambda\Lambda}^{16}\text{C}$  and  ${}_{\Xi}^{16}\text{C}$  in the  $\Xi^-$  bound region considering the one-step mechanism,  $K^-p \rightarrow K^+\Xi^-$  via  $\Xi^-$  doorways caused by the  $\Xi N$ - $\Lambda\Lambda$  coupling in the nuclear ( $K^-$ ,  $K^+$ ) reaction, rather than the two-step mechanism as  $K^-p \rightarrow \pi^0\Lambda$  followed by  $\pi^0p \rightarrow K^+\Lambda$  [7, 8]. These different mechanisms are well separated kinematically. The forward cross section for the  $K^-p \rightarrow K^+\Xi^-$  elementary process is at its maximum at  $p_{K^-} = 1.8$ – $1.9$  GeV/c, whereas the  $K^-p \rightarrow \pi^0\Lambda$  reaction at  $p_{K^-} = 1.1$  GeV/c leads to the maximal cross section for the  $\pi^0p \rightarrow K^+\Lambda$  process. The present study is the first attempt to evaluate a production spectrum of the  $\Lambda\Lambda$ - $\Xi$  hypernucleus via the  $\Xi N$ - $\Lambda\Lambda$  coupling from the inclusive ( $K^-$ ,  $K^+$ ) reaction, and to extract the  $\Xi^-$  admixture probability in the  $\Lambda\Lambda$  hypernucleus from the spectrum. We also discuss a contribution of the two-step processes in the ( $K^-$ ,  $K^+$ ) reactions within the eikonal approximation.

## II. CALCULATIONS

Let us consider the DCX ( $K^-$ ,  $K^+$ ) reaction on the  $^{16}\text{O}$  target at 1.8 GeV/c within a DWIA and examine the production cross sections and wave functions of the doubly strange hypernucleus. To fully describe the one-step process via  $\Xi^-$  doorways, as shown in Fig. 1(b), we perform nuclear  $\Lambda\Lambda$ - $\Xi$  coupled-channel calculations [13, 14], which are assumed to effectively represent the coupling nature in omitting other  $\Lambda\Sigma$  and  $\Sigma\Sigma$  channels for simplicity. Here we employ a multichannel coupled wave function of the  $\Lambda\Lambda$ - $\Xi$  nuclear state for a total

spin  $J_B$  within a weak coupling basis. It is written as

$$\begin{aligned} |\Psi_{J_B}(\Lambda\Lambda\Xi^-{}^{16}\text{C})\rangle &= \sum_{JJ''j_1j_2} [[\Phi_J(^{14}\text{C}), \varphi_{j_1}^{(\Lambda)}(\mathbf{r}_{\Lambda_1})]_{J''}, \varphi_{j_2}^{(\Lambda)}(\mathbf{r}_{\Lambda_2})]_{J_B} \\ &+ \sum_{JJ'j_pj_3} [\Phi_{J'}(^{15}\text{N}), \varphi_{j_3}^{(\Xi^-)}(\mathbf{r}_{\Xi})]_{J_B} \end{aligned} \quad (1)$$

with  $\Phi_{J'}(^{15}\text{N}) = \mathcal{A}[\Phi_J(^{14}\text{C}), \varphi_{j_p}^{(p)}(\mathbf{r}_p)]_{J'}$ , where  $\mathbf{r}_{\Lambda_1}$  ( $\mathbf{r}_p$ ) denotes the relative coordinate between the  $^{14}\text{C}$  core-nucleus and the  $\Lambda$  (proton), and  $\mathbf{r}_{\Lambda_2}$  ( $\mathbf{r}_{\Xi}$ ) denotes the relative coordinate between the center of mass of the  $^{14}\text{C}$ - $\Lambda$  ( $^{15}\text{N}$ ) subsystem and the  $\Lambda$  ( $\Xi^-$ ). Thus  $\varphi_{j_1,2}^{(\Lambda)}$ ,  $\varphi_{j_3}^{(\Xi^-)}$  and  $\varphi_{j_p}^{(p)}$  describe the relative wave functions of shell model states (that occupy  $j_{1,2}$ ,  $j_3$  and  $j_p$  orbits) for the  $\Lambda$ ,  $\Xi^-$  and proton, respectively;  $\Phi_J(^{14}\text{C})$  is a wave function of the  $^{14}\text{C}$  core-nucleus state, and  $\mathcal{A}$  is the anti-symmetrized operator for nucleons. The energy difference between  $^{15}\text{N}+\Xi^-$  and  $^{14}\text{C}+\Lambda+\Lambda$  channels is  $\Delta M = M(^{15}\text{N})+m_{\Xi^-}-M(^{14}\text{C})-2m_{\Lambda} = 18.4$  MeV, where  $M(^{15}\text{N})$ ,  $M(^{14}\text{C})$ ,  $m_{\Xi^-}$  and  $m_{\Lambda}$  are masses of the  $^{15}\text{N}$  nucleus, the  $^{14}\text{C}$  nucleus, the  $\Xi^-$  and  $\Lambda$  hyperons, respectively. We take the  $^{15}\text{N}$  core-nucleus states with  $J^\pi = 1/2^-(\text{g.s.})$  and  $3/2^-(6.32 \text{ MeV})$ , and the  $^{14}\text{C}$  core-nucleus states with  $J^\pi = 0^+(\text{g.s.})$  and  $2^+(7.01 \text{ MeV})$  that are given in  $(0p_{1/2}^{-1}0p_{1/2}^{-1})_{0+}$ ,  $(0p_{3/2}^{-1}0p_{1/2}^{-1})_{2+}$  and  $(0p_{3/2}^{-1}0p_{3/2}^{-1})_{0+,2+}$  configurations on  $^{16}\text{O}(\text{g.s.})$  [7, 8]. Because we assume only natural-parity  $\pi = (-1)^{J_B}$  states via  $\Xi^-$  doorways that are selectively formed by non-spin-flip processes in the forward  $K^-p \rightarrow K^+\Xi^-$  reaction, we consider a spin  $S = 0$ ,  $\Lambda\Lambda$  pair in the hypernucleus. If the  $\Lambda\Lambda$  component is dominant in a bound state, we can identify it as a state of the  $\Lambda\Lambda$  hypernucleus  ${}_{\Lambda\Lambda}^{16}\text{C}$ , in which the  $\Xi^-$  admixture probability can be estimated by

$$P_{\Xi^-} = \sum_{j_pj_3} \langle \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} | \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \rangle, \quad (2)$$

under the normalization of  $\sum_{j_1j_2} \langle \varphi_{j_1}^{(\Lambda)} \varphi_{j_2}^{(\Lambda)} | \varphi_{j_1}^{(\Lambda)} \varphi_{j_2}^{(\Lambda)} \rangle + \sum_{j_pj_3} \langle \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} | \varphi_{j_p}^{(p)} \varphi_{j_3}^{(\Xi^-)} \rangle = 1$ .

After we set up the  ${}_{\Lambda}^{15}\text{C}$  and  $^{15}\text{N}$  configurations in our model space with Eq. (1), we calculate the wave functions of  $\varphi_{j_2}^{(\Lambda)}(\mathbf{r}_{\Lambda_2})$  and  $\varphi_{j_3}^{(\Xi^-)}(\mathbf{r}_{\Xi})$  taking into account their channel coupling. Thus, the complete Green's function  $\mathbf{G}(\omega)$  [19] describes all information concerning  $({}_{\Lambda}^{15}\text{C} \otimes \Lambda) + ({}^{15}\text{N} \otimes \Xi^-)$  coupled-channel dynamics, as a function of the energy transfer  $\omega$ . It is numerically obtained as a solution of the  $N$ -channels radial coupled equations with a hyperon-nucleus potential  $\mathbf{U}$  [20, 21], which is written in an abbreviated notation as

$$\mathbf{G}(\omega) = \mathbf{G}^{(0)}(\omega) + \mathbf{G}^{(0)}(\omega)\mathbf{U}\mathbf{G}(\omega) \quad (3)$$

with

$$\mathbf{G}(\omega) = \begin{pmatrix} G_\Lambda(\omega) & G_X(\omega) \\ G_X(\omega) & G_\Xi(\omega) \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} U_\Lambda & U_X \\ U_X & U_\Xi \end{pmatrix}, \quad (4)$$

where  $\mathbf{G}^{(0)}(\omega)$  is a free Green's function. In our calculations, for example, we deal with  $N = 28$  for the  $J_B^\pi = 1^-$  state. The nuclear optical potentials  $U_Y$  ( $Y = \Xi$  or  $\Lambda$ ) can be written as

$$U_Y(r) = V_Y f(r, R, a) + iW_Y f(r, R', a') + iW_Y^{(D)} g(r, R', a'), \quad (5)$$

where  $f$  is the Woods-Saxon (WS) form,  $f(r, R, a) = [1 + \exp((r - R)/a)]^{-1}$ , and  $g$  is the derivative of the WS form,  $g(r, R', a') = -4a'(d/dr)f(r, R', a')$ . The spin-orbit potentials are neglected. In  $^{15}\text{N}-\Xi^-$  channels, we assume the strength parameter of  $V_\Xi = -24$  or  $-14$  MeV with  $a = 0.6$  fm and  $R = 1.10(A - 1)^{1/3} = 2.71$  fm in  $U_\Xi(r)$  [2, 5, 6], taking into account the Coulomb potential with the nuclear finite size  $R_C = 1.25A^{1/3} = 3.15$  fm [22]. The spreading imaginary potential in Eq. (5),  $\text{Im } U_Y$ , expresses complicated excited states via  $\Xi^- N \rightarrow \Lambda\Lambda$  conversion processes in  $_{\Xi^-}^{16}\text{C}$  or  $_{\Lambda\Lambda}^{16}\text{C}$  above the  $_{\Lambda\Lambda}^{15}\text{C} + n$  threshold at 8.2 MeV, as a function of the excitation energy  $E_{\text{ex}}$  measured from an energy of the  $_{\Lambda\Lambda}^{16}\text{C}$  ground state, as often used in nuclear optical models. Since we have no criterion for a choice of  $W_\Xi$  or  $W_\Xi^{(D)}$  in the limited experimental data, we adjust appropriately the strength parameter of  $W_\Xi$  in the WS-type to give widths of  $\Xi^-$  quasibound states in recent calculations [5, 6, 23]. In  $^{14}\text{C}-\Lambda\Lambda$  channels, we should use a  $_{\Lambda}^{15}\text{C}-\Lambda$  potential, which can be constructed in folded potential models [24]:

$$U_\Lambda(r) = \int \rho_{J''}(\mathbf{r}_\Lambda) [U_{C\Lambda}(|\mathbf{r} + \lambda_\Lambda \mathbf{r}_\Lambda|) + V_{\Lambda\Lambda}(|\mathbf{r} - \nu_\Lambda \mathbf{r}_\Lambda|)] d\mathbf{r}_\Lambda, \quad (6)$$

where  $\rho_{J''}(\mathbf{r}_\Lambda) = \sum_{j_1 m_1} \langle JM j_1 m_1 | J'' M'' \rangle^2 |\varphi_{j_1}^{(\Lambda)}(\mathbf{r}_\Lambda)|^2$  and  $\lambda_\Lambda = 1 - \nu_\Lambda = m_\Lambda / (M(^{14}\text{C}) + m_\Lambda)$ .  $U_{C\Lambda}$  and  $V_{\Lambda\Lambda}$  denote an optical potential for  $^{14}\text{C}-\Lambda$  as given in Eq. (5) and a  $\Lambda\Lambda$  residual interaction, respectively. Here we neglected  $V_{\Lambda\Lambda}$  for simplicity. The real part of  $U_{C\Lambda}$  leads to  $B_\Lambda = 12.2$  MeV for the  $(0s)_\Lambda$  state and  $B_\Lambda = 1.6$  MeV for the  $(0p)_\Lambda$  state in  $_{\Lambda}^{15}\text{C}$  [25], and its imaginary part exhibits a flux loss of the wave functions through the core excitations of  $^{14}\text{C}^*$ . We assume  $W_\Lambda \simeq \frac{1}{4}W_N$  and  $W_\Lambda^{(D)} \simeq \frac{1}{4}W_N^{(D)}$  where parameters of  $W_N$  and  $W_N^{(D)}$  for nucleon were obtained in Ref. [26] because the well depth of the imaginary potential for  $\Lambda$  is by a factor of 4 weaker than that for nucleon in  $g$ -matrix calculations [27].

The  $\Lambda\Lambda-\Xi$  coupling potential  $U_X$  in off-diagonal parts of  $\mathbf{U}$  is the most interesting object in this calculation [10–16]. It can be obtained by a two-body  $\Xi N-\Lambda\Lambda$  potential  $v_{\Xi N, \Lambda\Lambda}(\mathbf{r}', \mathbf{r})$  with the  $^1S_0$ , isospin  $T = 0$  state. Here we use a zero-range interaction  $v_{\Xi N, \Lambda\Lambda}(\mathbf{r}', \mathbf{r}) =$

$v_{\Xi N, \Lambda\Lambda}^0 \delta_{S,0} \delta(\mathbf{r}' - \mathbf{r})$  in a real potential for simplicity, where  $v_{\Xi N, \Lambda\Lambda}^0$  is the strength parameter that should be connected with volume integral  $\int v_{\Xi N, \Lambda\Lambda}(\mathbf{r}) d\mathbf{r} = v_{\Xi N, \Lambda\Lambda}^0$  [13, 14, 16]. Thus the matrix elements can be easily estimated by use of Racah algebra [29]:

$$\begin{aligned} U_X(r) &= \left\langle [\Phi_{J'}(^{15}\text{N}) \otimes \mathcal{Y}_{j'\ell's'}^{(\Xi^-)}(\hat{\mathbf{r}})]_{J_B} \middle| \sum_i v_{\Xi N, \Lambda\Lambda}(\nu_i \mathbf{r}'_i, \mathbf{r}) \right. \\ &\quad \left. \times \middle| [\Phi_J(^{14}\text{C}), \varphi_{j_1}^{(\Lambda)}]_{J''} \otimes \mathcal{Y}_{j\ell s}^{(\Lambda)}(\hat{\mathbf{r}})]_{J_B} \right\rangle \\ &= \sum_{LSK} \sqrt{1/2} v_{\Xi N, \Lambda\Lambda}^0 \delta_{S,0} C_{LSK}^{J_B}(J' J'') \mathcal{F}_{LSK}^{J' J''}(r), \end{aligned} \quad (7)$$

where  $\mathcal{Y}_{j\ell s} = [Y_\ell \otimes X_{\frac{1}{2}}]_j$  is a spin-orbit function and  $C_{LSK}^{J_B}(J' J'')$  is a purely geometrical factor [29];  $\mathcal{F}_{LSK}^{J' J''}(r)$  is the nuclear form factor including a recoupling coefficient of  $U(J j_1 J'' K; J' j_p)$  [16], a parentage coefficient for proton removal from  $^{15}\text{N}(1/2^-, 3/2^-)$  [30] and the center-of-mass correction of a factor  $\sqrt{A/(A-1)}$  [31]. The factor  $\sqrt{1/2}$  comes from the procedure handling a transition between  $p\Xi^-$  and  $\Lambda\Lambda$  states in the nucleus.

The inclusive  $K^+$  double-differential laboratory cross section of the  $\Lambda\Lambda-\Xi$  production in the nuclear  $(K^-, K^+)$  reaction can be written within the DWIA [32, 33] using the Green's function method [19]. In the one-step mechanism,  $K^- p \rightarrow K^+ \Xi^-$  via  $\Xi^-$  doorways, it is given [21] as

$$\begin{aligned} &\left( \frac{d^2\sigma}{d\Omega_K dE_K} \right)_{\text{lab}} \\ &= \beta \frac{1}{[J_A]} \sum_{M_z} \sum_{\alpha' \alpha} \left( -\frac{1}{\pi} \right) \text{Im} \left[ \int d\mathbf{r}' d\mathbf{r} F_{\Xi}^{\alpha' \dagger}(\mathbf{r}') \right. \\ &\quad \left. \times G_{\Xi}^{\alpha' \alpha}(\omega, \mathbf{r}', \mathbf{r}) F_{\Xi}^{\alpha}(\mathbf{r}) \right] \end{aligned} \quad (8)$$

for the target with a spin  $J_A$  and its  $z$ -component  $M_z$ , where  $[J_A] = 2J_A + 1$ , and a kinematical factor  $\beta$  [34] that expresses the translation from the two-body  $K^-p$  laboratory system to the  $K^- - ^{16}\text{O}$  laboratory system [2]. The production amplitude  $F_{\Xi}^{\alpha}$  is

$$\begin{aligned} F_{\Xi}^{\alpha}(\mathbf{r}) &= \bar{f}_{K^- p \rightarrow K^+ \Xi^-} \chi_{\mathbf{p}_{K^+}}^{(-)*} \left( \frac{M_C}{M_B} \mathbf{r} \right) \chi_{\mathbf{p}_{K^-}}^{(+)} \left( \frac{M_C}{M_A} \mathbf{r} \right) \\ &\quad \times \langle \alpha | \hat{\psi}_p(\mathbf{r}) | \Psi_{J_A M_z} \rangle, \end{aligned} \quad (9)$$

where  $\bar{f}_{K^- p \rightarrow K^+ \Xi^-}$  is a Fermi-averaged amplitude for the  $K^- p \rightarrow K^+ \Xi^-$  reaction in nuclear medium [2], and  $\chi_{\mathbf{p}_{K^+}}^{(-)}$  and  $\chi_{\mathbf{p}_{K^-}}^{(+)}$  are the distorted waves for outgoing  $K^+$  and incoming  $K^-$  mesons, respectively; the factors of  $M_C/M_B$  and  $M_C/M_A$  take into account the recoil effects, where  $M_A$ ,  $M_B$  and  $M_C$  are masses of the target, the final state and the core-nucleus,

respectively.  $\langle \alpha | \hat{\psi}_p | \Psi_{J_A M_z} \rangle$  is a hole-state wave function for a struck proton in the target, where  $\alpha$  denotes the complete set of eigenstates for the system. It should be recognized that the  $\Lambda\Lambda$ - $\Xi$  coupled-channel Green's function with the spreading potential provides an advantage of estimating contributions from sources both as  $\Lambda\Lambda$  components in  $\Xi^-$ -nucleus eigenstates [16] and as  $\Xi^-p \rightarrow \Lambda\Lambda$  quasi-scattering processes in the nucleus [15].

Because the momentum transfer is very high in the nuclear ( $K^-$ ,  $K^+$ ) reaction at 1.8 GeV/c, i.e.,  $q_{\Xi^-} \simeq 360$ –430 MeV/c, the distorted waves for outgoing  $K^+$  and incoming  $K^-$  in Eq. (9) are calculated with the help of the eikonal approximation [32, 35]. As the distortion parameters, we use total cross sections of  $\sigma_{K^-N} = 28.9$  mb for  $K^-N$  scattering and  $\sigma_{K^+N} = 19.4$  mb for  $K^+N$  scattering [6], and  $\alpha_{K^-N} = \alpha_{K^+N} = 0$ . We take 35  $\mu\text{b/sr}$  as the laboratory cross section of  $d\sigma/d\Omega = \bar{\alpha} |\bar{f}_{K^-p \rightarrow K^+\Xi^-}|^2$  including the kinematical factor  $\bar{\alpha}$  [5, 9]. For the target nucleus  $^{16}\text{O}$  with  $J_A^\pi = 0^+$ , we assume the wave functions for the proton hole-states in the relative coordinate, which are calculated with central (WS-type) and spin-orbit potentials [22], by fitting to the charge rms radius of 2.72 fm [36]. For the energies (widths) for proton-hole states, we input 12.1 (0.0), 18.4 (2.5) and 36 (10) MeV for  $0p_{1/2}^{-1}$ ,  $0p_{3/2}^{-1}$  and  $0s_{1/2}^{-1}$  states, respectively.

Three parameters,  $V_\Xi$ ,  $W_\Xi$  and  $v_{\Xi N, \Lambda\Lambda}^0$ , are very important for calculating the inclusive spectra with the one-step mechanism. These parameters are strongly connected each other for the shape of the spectrum and its magnitude, as well as for the  $\Xi^-$  binding energies and widths of the  $\Xi^-$  states. Several authors [10, 14, 16] investigated the effects of the  $\Xi N$ - $\Lambda\Lambda$  coupling in light nuclei evaluating the volume integrals for  $k_F$ -dependent  $\Xi N$ - $\Lambda\Lambda$  effective interactions based on Nijmegen potentials [28], in which these values are strongly model dependent; for example, 250.9, 370.2, 501.5, 582.1 and 873.9 MeV $\cdot\text{fm}^3$  for NHC-D, NSC97e, NSC04a, NHC-F and NSC04d potentials ( $k_F = 1.0 \text{ fm}^{-1}$ ), respectively [14, 28]. The  $\Xi^-p \rightarrow \Lambda\Lambda$  conversion cross section of  $(v\sigma)_{\Xi^-p \rightarrow \Lambda\Lambda} \simeq 7.9$  mb also yields to be about 544 MeV $\cdot\text{fm}^3$  [16]. To see the dependence of the spectrum on the  $\Xi N$ - $\Lambda\Lambda$  coupling strength, here, we choose typical values of  $v_{\Xi N, \Lambda\Lambda}^0 = 250$  and 500 MeV, which approximate the volume integrals of NHC-D and NSC04a, respectively. We take the spreading potential of  $\text{Im } U_\Xi$  to be  $W_\Xi \simeq -3$  MeV at the  $^{15}\text{N} + \Xi^-$  threshold [5, 6, 14, 18]. It should be noticed that this spreading potential expresses nuclear core breakup processes caused by the  $\Xi^-p \rightarrow \Lambda\Lambda$  conversion in the  $^{15}\text{N}$  nucleus, and its effect cannot be involved in  $U_X$ .



### III. RESULTS AND DISCUSSION

Now let us discuss the inclusive spectrum in the  $^{16}\text{O}(K^-, K^+)$  reaction at 1.8 GeV/c ( $0^\circ$ ) in order to examine the dependence of the spectrum on the parameters of  $V_{\Xi}$  and  $v_{\Xi N, \Lambda\Lambda}^0$ . We consider contributions of the  $\Lambda\Lambda$ - $\Xi$  nuclear bound and resonance states to the  $\Xi^- p \rightarrow \Lambda\Lambda$  conversion processes in the  $\Xi^-$  bound region.

In Fig. 2(a), we show the calculated spectra in the  $\Xi^-$  bound region without the  $\Lambda\Lambda$ - $\Xi$  coupling potential when we use  $V_{\Xi} = -24$  MeV or  $-14$  MeV with the Coulomb potential. The calculated spectra are in agreement with the spectra obtained by previous works [6]. In the case of  $V_{\Xi} = -24$  MeV, we find that a broad peak of the  $[^{15}\text{N}(1/2^-) \otimes s_{\Xi^-}]_{1^-}$  quasibound state in  $^{16}_{\Xi^-}\text{C}$  is located at  $B_{\Xi^-} = 13.4$  MeV with a sizable width of  $\Gamma = 3.5$  MeV, and a clear peak of the  $[^{15}\text{N}(1/2^-) \otimes p_{\Xi^-}]_{2^+}$  quasibound state at  $B_{\Xi^-} = 3.7$  MeV with  $\Gamma = 3.1$  MeV. Integrated cross sections indicate  $d\sigma(0^\circ)/d\Omega \simeq 28$  nb/sr for the  $1^-$  state and 77 nb/sr for the  $2^+$  state in  $^{16}_{\Xi^-}\text{C}$ . In the case of  $V_{\Xi} = -14$  MeV, which is favored in recent calculations [6, 13, 14, 18], we have the  $[^{15}\text{N}(1/2^-) \otimes s_{\Xi^-}]_{1^-}$  state at  $B_{\Xi^-} = 6.8$  MeV with  $\Gamma = 3.8$  MeV and the  $[^{15}\text{N}(1/2^-) \otimes p_{\Xi^-}]_{2^+}$  at  $B_{\Xi^-} = 0.5$  MeV with  $\Gamma = 1.1$  MeV. The integrated cross sections indicate  $d\sigma(0^\circ)/d\Omega \simeq 6$  nb/sr for the  $1^-$  state and 9 nb/sr for the  $2^+$  state. Note that the  $\Xi^- p \rightarrow \Lambda\Lambda$  conversion processes that can be described by the absorption potential  $\text{Im} U_{\Xi}$ , must appear above the  $^{15}_{\Lambda\Lambda}\text{C} + n$  decay threshold at  $\omega = 360.4$  MeV (which corresponds to  $B_{\Lambda\Lambda} = 16.7$  MeV). We confirm that no clear signal of the  $\Xi^-$  bound state is measured if  $V_{\Xi}$  is shallow such as  $-V_{\Xi} \leq 14$  MeV and/or  $W_{\Xi}$  is sizably absorptive ( $-W_{\Xi} \geq 3$  MeV at the  $^{15}\text{N} + \Xi^-$  threshold) in  $U_{\Xi}$ . Nevertheless, the production of these  $\Xi^-$  states as well as  $\Xi^-$  states coupled to a  $^{15}\text{N}(3/2^-)$  nucleus is essential in this model because these states act as doorways when we consider the  $\Lambda\Lambda$  states formed in the one-step mechanism. We also expect to extract properties of the  $\Xi$ -nucleus potential such as  $V_{\Xi}$  and  $W_{\Xi}$  from the  $\Xi^-$  continuum spectra in the  $(K^-, K^+)$  reactions on nuclear targets, as already discussed for studies of the  $\Sigma^-$ -nucleus potential in nuclear  $(\pi^-, K^+)$  reactions [38, 39].

———— **FIG. 2** ————

On the other hand, the  $\Lambda\Lambda$ - $\Xi$  coupling plays an important role in making a production of the  $\Lambda\Lambda$  states via  $\Xi^-$  doorways below the  $^{15}\text{N} + \Xi^-$  threshold. The positions of their peaks must be slightly shifted downward by the energy shifts  $\Delta B_{\Lambda\Lambda}$  due to the coupling potential



in Eq. (7). When  $v_{\Xi N, \Lambda\Lambda}^0 = 500$  MeV (250 MeV), we obtain  $\Delta B_{\Lambda\Lambda} = 1.17$  MeV (0.15 MeV) and the  $\Xi^-$  admixture probability  $P_{\Xi^-} = 5.24\%$  (0.87%) in the  $[^{14}\text{C}(0^+) \otimes s_{\Lambda}p_{\Lambda}]_{1^-}$  excited state and  $\Delta B_{\Lambda\Lambda} = 0.38$  MeV (0.09 MeV) and  $P_{\Xi^-} = 0.58\%$  (0.14%) in the  $[^{14}\text{C}(0^+) \otimes s_{\Lambda}^2]_{0^+}$  ground state. The value of  $P_{\Xi^-}$  in the  $1^-$  state is by a factor of 6–9 as large as that in the  $0^+$  state. These values are strongly connected with the magnitude of the peak for the  $\Lambda\Lambda$  state in the spectrum.

In Fig. 2(b), we show the calculated spectra with the  $\Lambda\Lambda$ – $\Xi$  coupling potential when  $V_{\Xi} = -14$  MeV. We recognize that the shape of these spectra is quite sensitive to the value of  $v_{\Xi N, \Lambda\Lambda}^0$ , and it is obvious that no  $\Xi N$ – $\Lambda\Lambda$  coupling cannot describe the spectrum of the  $\Lambda\Lambda$  states below the  $^{14}\text{C} + \Lambda + \Lambda$  threshold. The calculated spectrum for  $v_{\Xi N, \Lambda\Lambda}^0 = 500$  MeV has a fine structure of the  $\Lambda\Lambda$  excited states in  $^{16}_{\Lambda\Lambda}\text{C}$ . We find that significant peaks of the  $1^-$  excited states with  $^{14}\text{C}(0^+) \otimes s_{\Lambda}p_{\Lambda}$  at  $\omega = 362.1$  MeV ( $B_{\Lambda\Lambda} = 15.1$  MeV) and  $^{14}\text{C}(2^+) \otimes s_{\Lambda}p_{\Lambda}$  at  $\omega = 368.5$  MeV ( $B_{\Lambda\Lambda} = 8.7$  MeV), and small peaks of the  $2^+$  excited states with  $^{14}\text{C}(0^+) \otimes p_{\Lambda}^2$  at  $\omega = 373.8$  MeV ( $B_{\Lambda\Lambda} = 3.4$  MeV) and  $^{14}\text{C}(2^+) \otimes p_{\Lambda}^2$  at  $\omega = 380.4$  MeV ( $B_{\Lambda\Lambda} = -3.2$  MeV). This result arises from the fact that the high momentum transfer  $q_{\Xi^-} \simeq 400$  MeV/c leads to a preferential population of the spin-stretched  $\Xi^-$  doorway states followed by the  $[^{15}\text{N}(1/2^-, 3/2^-) \otimes s_{\Xi^-}]_{1^-} \rightarrow [^{14}\text{C}(0^+, 2^+) \otimes s_{\Lambda}p_{\Lambda}]_{1^-}$  and  $[^{15}\text{N}(1/2^-, 3/2^-) \otimes p_{\Xi^-}]_{2^+} \rightarrow [^{14}\text{C}(0^+, 2^+) \otimes p_{\Lambda}^2]_{2^+}$  transitions, to which a sum of their continuum states may contribute predominately in the  $(K^-, K^+)$  reactions. Figure 3 also displays partial-wave decomposition of the calculated inclusive spectrum for  $^{16}_{\Lambda\Lambda}\text{C}$  in the  $\Lambda\Lambda$  bound region when  $V_{\Xi} = -14$  MeV and  $v_{\Xi N, \Lambda\Lambda}^0 = 500$  MeV. The integrated cross sections at  $\theta_{\text{lab}} = 0^\circ$  for the  $1^-$  excited states with  $^{14}\text{C}(0^+) \otimes s_{\Lambda}p_{\Lambda}$  and  $^{14}\text{C}(2^+) \otimes s_{\Lambda}p_{\Lambda}$  are respectively

$$\frac{d\sigma}{d\Omega_L} [^{16}_{\Lambda\Lambda}\text{C}(1^-)] \simeq 7 \text{ nb/sr and } 12 \text{ nb/sr}, \quad (10)$$

where the  $\Xi^-$  admixture probabilities of these states amount to  $P_{\Xi^-} = 5.2\%$  and  $8.8\%$ , respectively. It should be noticed that the cross sections are on the same order of magnitude as those for the  $1^-$  and  $2^+$  quasibound states that are located at  $B_{\Xi^-} = 6.8$  MeV and  $0.5$  MeV, respectively, in the  $^{16}_{\Xi^-}\text{C}$  hypernucleus. Therefore, such  $\Lambda\Lambda$  excited states below the  $^{14}\text{C} + \Lambda + \Lambda$  threshold will be measured experimentally at the J-PARC facilities [3].

### — FIG. 3 —

On the other hand, it is extremely difficult to populate the  $0^+$  ground state with  $^{14}\text{C}(0^+) \otimes s_{\Lambda}^2$  at  $\omega \simeq 352.3$  MeV ( $B_{\Lambda\Lambda} \simeq 24.9$  MeV) and also the  $2^+$  excited state with  $^{14}\text{C}(2^+) \otimes s_{\Lambda}^2$

at  $\omega \simeq 359.6$  MeV ( $B_{\Lambda\Lambda} \simeq 17.5$  MeV) in the one-step mechanism via  $\Xi^-$  doorways in the  $(K^-, K^+)$  reactions. The high momentum transfer of  $q_{\Xi} \simeq 400$  MeV/c necessarily leads to the non-observability with  $\Delta L = 0$ . Thus the integrated cross section of the  $0^+$  state is found to be about 0.02 nb/sr, of which the  $q$  dependence is approximately governed by a factor of  $\exp(-\frac{1}{2}(\tilde{b}q_{\Xi})^2)$  where a size parameter  $\tilde{b} = 1.84$  fm. There is no production in the  $2^+$  state with  $^{14}\text{C}(2^+) \otimes s_{\Lambda}^2$  under the angular-momentum conservation in the  $^{16}\text{O}(K^-, K^+)$  reactions by the one-step mechanism. The contribution of these states to the  $\Lambda\Lambda$  spectrum in the one-step mechanism is completely different from that in the two-step mechanism as obtained in Refs. [7, 8].

In the  $(K^-, K^+)$  reaction,  $\Lambda\Lambda$  hypernuclear states can be also populated by the two-step mechanism,  $K^-p \rightarrow \pi^0\Lambda$  followed by  $\pi^0p \rightarrow K^+\Lambda$  [7–9], as shown in Fig. 1(a). Following the procedure by Dover [7, 9], a crude estimate can be obtained for the contribution of this two-step processes in the eikonal approximation using a harmonic oscillator model. The cross section at  $0^\circ$  for quasielastic  $\Lambda\Lambda$  production at  $p_{K^-} = 1.8$  GeV/c in the two-step mechanism, which is summed over all final state, is given [9] as

$$\begin{aligned} \sum_f \left( \frac{d\sigma_f^{(2)}}{d\Omega_L} \right)_{0^\circ} &\approx \frac{2\pi\xi}{p_\pi^2} \left\langle \frac{1}{r^2} \right\rangle \left( \alpha \frac{d\sigma}{d\Omega_L} \right)_{0^\circ}^{K^-p \rightarrow \pi^0\Lambda} \\ &\times \left( \alpha \frac{d\sigma}{d\Omega_L} \right)_{0^\circ}^{\pi^0p \rightarrow K^+\Lambda} N_{\text{eff}}^{pp}, \end{aligned} \quad (11)$$

where  $\xi = 0.022\text{--}0.019$  mb $^{-1}$  is a constant nature of the angular distributions of the two elementary processes,  $p_\pi \simeq 1.68$  GeV/c is the intermediate pion momentum, and  $\langle 1/r^2 \rangle \simeq 0.028$  mb $^{-1}$  is the mean inverse-square radial separation of the proton pair.  $N_{\text{eff}}^{pp} \simeq 1$  is the effective number of proton pairs including the nuclear distortion effects [7]. The elementary laboratory cross section  $(\alpha d\sigma/d\Omega_L)_{0^\circ}$  is estimated to be 1.57–1.26 mb/sr for  $K^-p \rightarrow \pi^0\Lambda$  and 0.070–0.067 mb/sr for  $\pi^0p \rightarrow K^+\Lambda$  depending on the nuclear medium corrections. This yields

$$\sum_f \left( \frac{d\sigma_f^{(2)}}{d\Omega_L} \right)_{0^\circ} \simeq 0.06\text{--}0.04 \text{ } \mu\text{b/sr}, \quad (12)$$

which is half smaller than  $\sim 0.14$   $\mu\text{b/sr}$  at 1.1 GeV/c. Considering a high momentum transfer  $q \simeq 400$  MeV/c in the  $(K^-, K^+)$  reactions, we expect that the production probability for the  $\Lambda\Lambda$  bound states does not exceed 1% in the quasielastic  $\Lambda\Lambda$  production, so that an estimate of the  $\Lambda\Lambda$  hypernucleus in the two-step mechanism may be on the order of 0.1–1 nb/sr. This

cross section is smaller than the cross section for the  $\Lambda\Lambda$   $1^-$  states we mentioned above in the one-step mechanism. Consequently, we believe that the one-step mechanism acts in a dominant process in the  $(K^-, K^+)$  reaction at 1.8 GeV/c ( $0^\circ$ ) when  $v_{\Xi N, \Lambda\Lambda}^0 = 400\text{--}600$  MeV. This implies that the  $(K^-, K^+)$  spectrum provides valuable information concerning  $\Xi N$ – $\Lambda\Lambda$  dynamics in the  $S = -2$  systems such as  $\Lambda\Lambda$  and  $\Xi$  hypernuclei, which are often discussed in a full coupling scheme [40].

#### IV. SUMMARY AND CONCLUSION

We have examined theoretically production of doubly strange hypernuclei in the DCX  $^{16}\text{O}(K^-, K^+)$  reaction at 1.8 GeV/c within DWIA calculations using coupled-channel Green's functions. We have shown that the  $\Xi^-$  admixture in the  $\Lambda\Lambda$  hypernuclei plays an essential role in producing the  $\Lambda\Lambda$  states in the  $(K^-, K^+)$  reaction.

In conclusion, the calculated spectrum for the  $^{16}_{\Xi^-}\text{C}$  and  $^{16}_{\Lambda\Lambda}\text{C}$  hypernuclei in the one-step mechanism  $K^-p \rightarrow K^+\Xi^-$  via  $\Xi^-$  doorways predicts promising peaks of the  $\Lambda\Lambda$  bound and excited states in the  $^{16}\text{O}(K^-, K^+)$  reactions at 1.8 GeV/c ( $0^\circ$ ). It has been shown that the integrated cross sections for the significant  $1^-$  excited states in  $^{16}_{\Lambda\Lambda}\text{C}$  are on the order of 7–12 nb/sr depending on the  $\Xi N$ – $\Lambda\Lambda$  coupling strength and also the attraction in the  $\Xi$ -nucleus potential. The  $\Xi^-$  admixture probabilities are on the order of 5–9%. The sensitivity to the potential parameters indicates that the nuclear  $(K^-, K^+)$  reactions have a high ability for the theoretical analysis of precise wave functions in the  $\Lambda\Lambda$  and  $\Xi$  hypernuclei. New information on  $\Lambda\Lambda$ – $\Xi$  dynamics in nuclei from the  $(K^-, K^+)$  data at J-PARC facilities [3] will bring the  $S = -2$  world development in nuclear physics.

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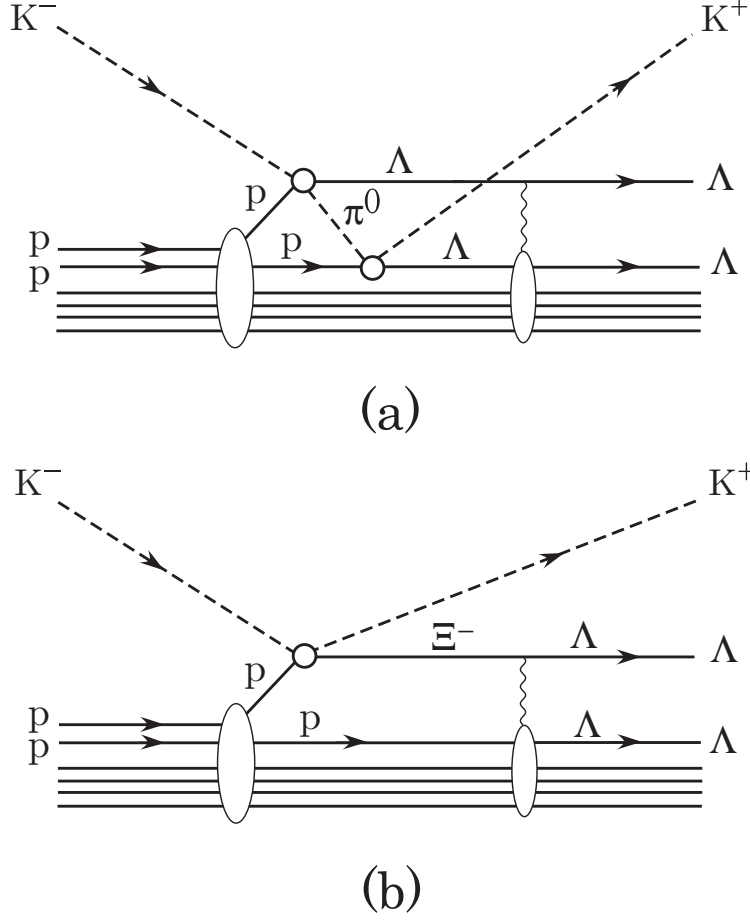


FIG. 1: Diagrams for DCX nuclear ( $K^-$ ,  $K^+$ ) reactions: (a) a two-step mechanism,  $K^-p \rightarrow \pi^0\Lambda$  followed by  $\pi^0p \rightarrow K^+\Lambda$ , and (b) a one-step mechanism,  $K^-p \rightarrow K^+\Xi^-$  via  $\Xi^-$  doorways caused by the  $\Xi^-p\text{-}\Lambda\Lambda$  coupling.

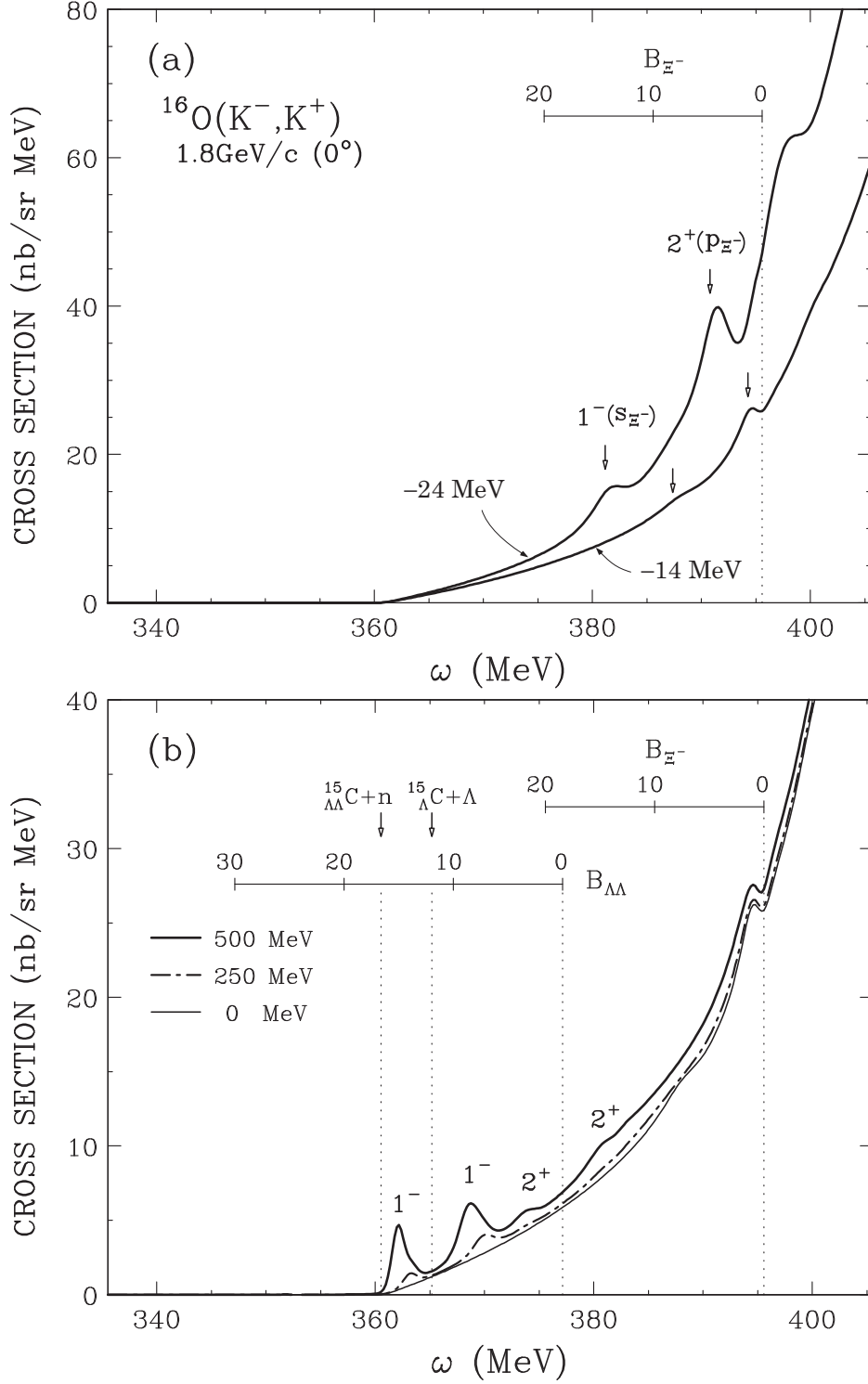


FIG. 2: Calculated inclusive  $\Lambda\Lambda\text{-}\Xi$  spectra by the one-step mechanism in the  $^{16}\text{O}(K^-, K^+)$  reaction at  $1.8\text{ GeV}/c$  ( $0^\circ$ ), with a detector resolution of  $1.5\text{ MeV FWHM}$ ; (a)  $V_{\Xi} = -24$  or  $-14\text{ MeV}$  without the  $\Lambda\Lambda\text{-}\Xi$  coupling potential. The  $\Xi$  conversion decay occurs above the  $^{15}_{\Lambda\Lambda}\text{C} + n$  threshold at  $\omega = 361\text{ MeV}$ ; (b)  $V_{\Xi} = -14\text{ MeV}$  with the  $\Lambda\Lambda\text{-}\Xi$  coupling potential obtained by  $v_{\Xi N, \Lambda\Lambda}^0 = 0, 250$  and  $500\text{ MeV}$ .



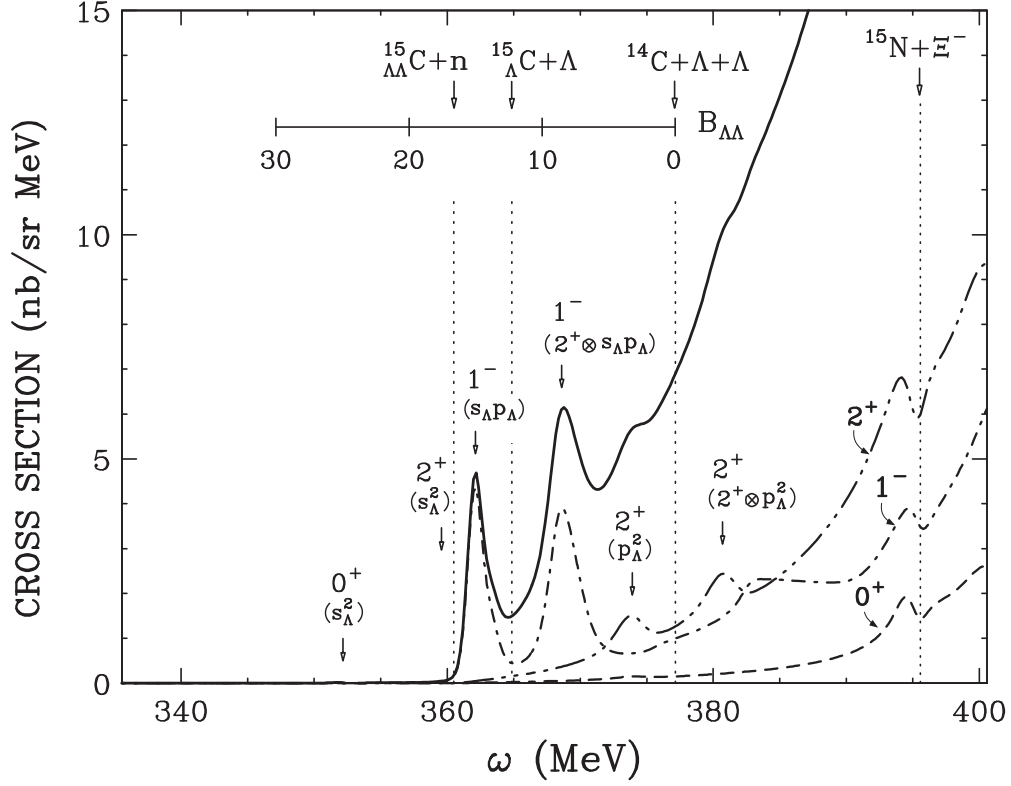


FIG. 3: Partial-wave decomposition of the calculated inclusive spectrum by the one-step mechanism near the  $^{14}\text{C}+\Lambda+\Lambda$  threshold in the  $^{16}\text{O}(K^-, K^+)$  reaction at 1.8 GeV/c ( $0^\circ$ ).  $V_{\Xi} = -14$  MeV and  $v_{\Xi N, \Lambda\Lambda}^0 = 500$  MeV were used. The labels  $0^+(s_{\Lambda}^2)$ ,  $1^-(s_{\Lambda}p_{\Lambda})$  and  $2^+(p_{\Lambda}^2)$  denote the  $J^\pi$   $\Lambda\Lambda$  nuclear states of  $(0s_{\Lambda})^2$ ,  $(0s_{\Lambda})(0p_{\Lambda})$  and  $(0p_{\Lambda})^2$  coupled with  $^{14}\text{C}(0^+)$ , respectively. The labels  $2^+(s_{\Lambda}^2)$ ,  $1^-(2^+ \otimes s_{\Lambda}p_{\Lambda})$  and  $2^+(2^+ \otimes p_{\Lambda}^2)$  denote the states of  $(0s_{\Lambda})^2$ ,  $(0s_{\Lambda})(0p_{\Lambda})$  and  $(0p_{\Lambda})^2$  coupled with  $^{14}\text{C}(2^+)$ , respectively.